Redistribution effects in angle and frequency, response functions…

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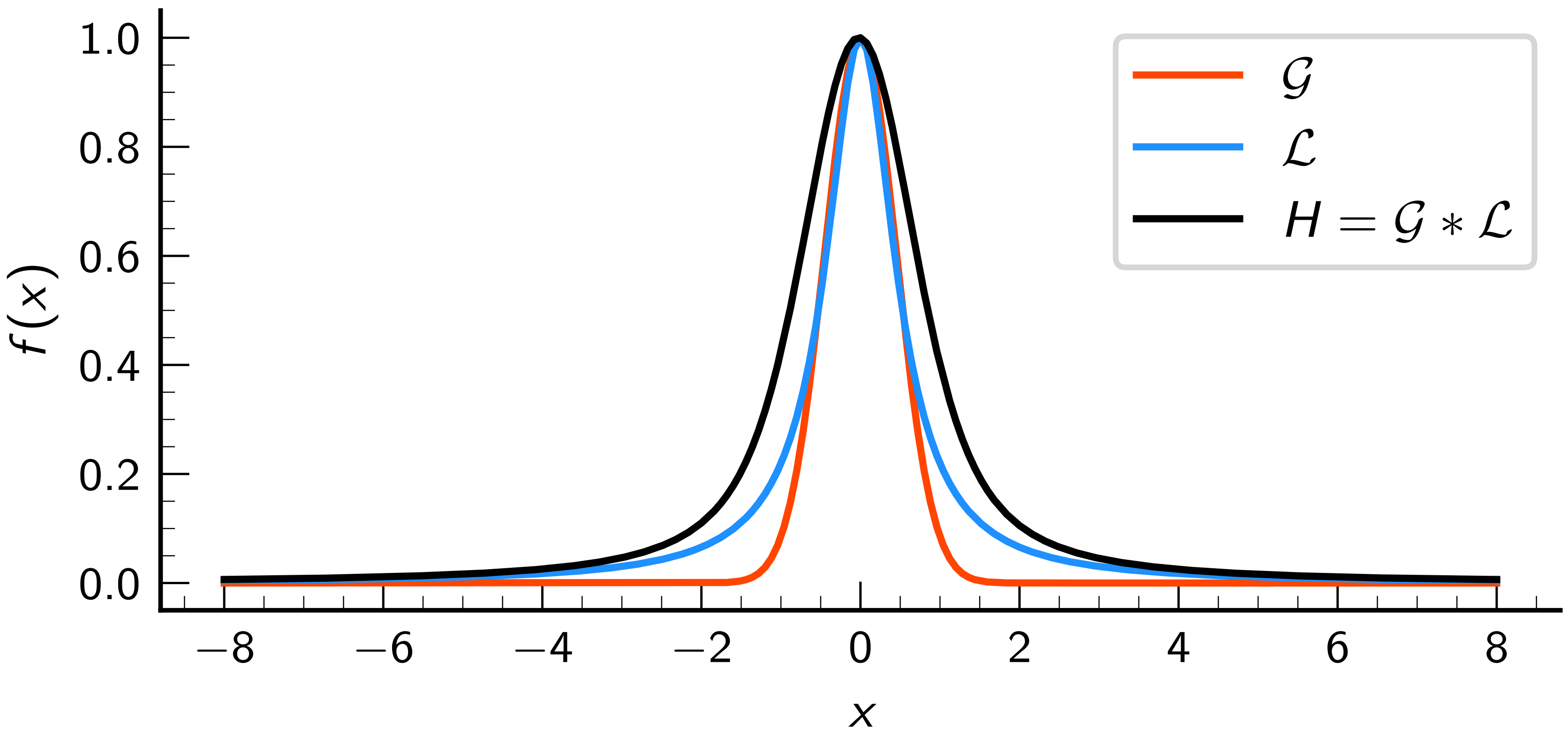
Line profiles

$$S_\nu^{\rm line} = \frac{j_\nu}{\alpha_\nu} = \frac{n_u A_{ul} \psi}{n_l B_{lu} \phi - n_u B_{ul} \chi}$$

We have a full chapter dedicated to line broadening and profiles in the class notes:

- **Level finite lifetime** (Heisenberg's principle): Lorentzian profile.
- **Collisions** with electrons, hydrogen atoms: Lorentzian profile.
- **Thermal motions** of atoms: Gaussian profile.
- **Turbulent (macroscopic) motions**: Gaussian profile.

We can combine these broadening effects by **convolution** of the profiles.
The convolution of a Lorentzian with a Gaussian is a **Voigt profile**.



Line profiles

$$\tau_\nu = \int_0^s \alpha_\nu(s) ds'$$

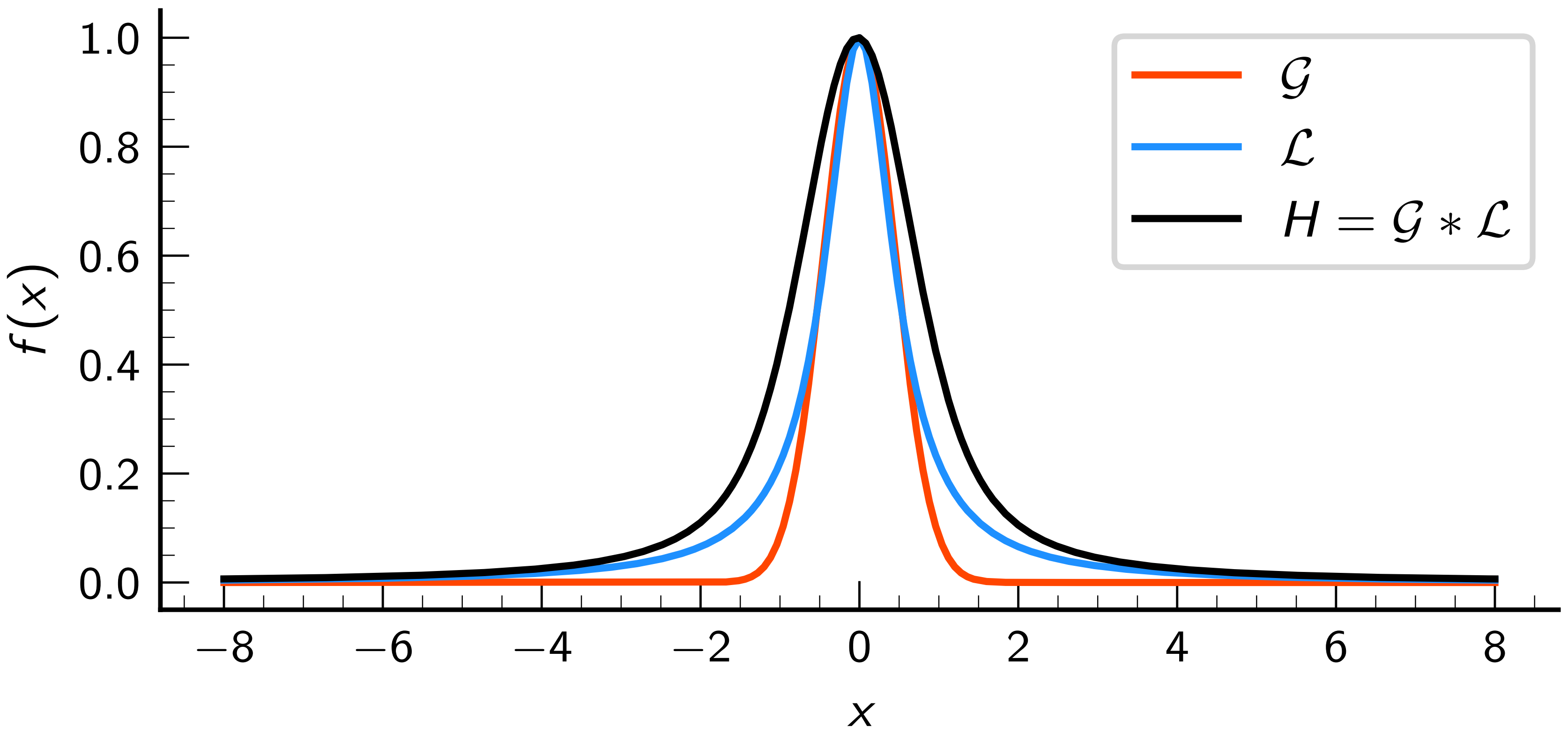
Let's assume a constant property slab so $\alpha_\nu(s) = \alpha_\nu^0 = \eta_0 \psi(\nu - \nu_0)$:

$$\tau_\nu = \eta_0 \psi(\nu - \nu_0) \cdot s$$

At line center $\psi = \psi_{\max} \approx 1 \longrightarrow \tau_\nu = \tau_{\max}$.

In wings: $0 \leq \tau_\nu < \tau_{\max}$

If $\tau_\nu < 1$ the medium is optically thin. If $\tau_\nu \gtrapprox 1$ the medium is optically thick.



Line profiles

Let's assume an unpolarized line profile with a linear source function:

$$I_\nu = S_0 + S_1 \frac{1}{1 + \eta_0 \psi(\nu - \nu_0)}$$

If the line is optically thin, then:

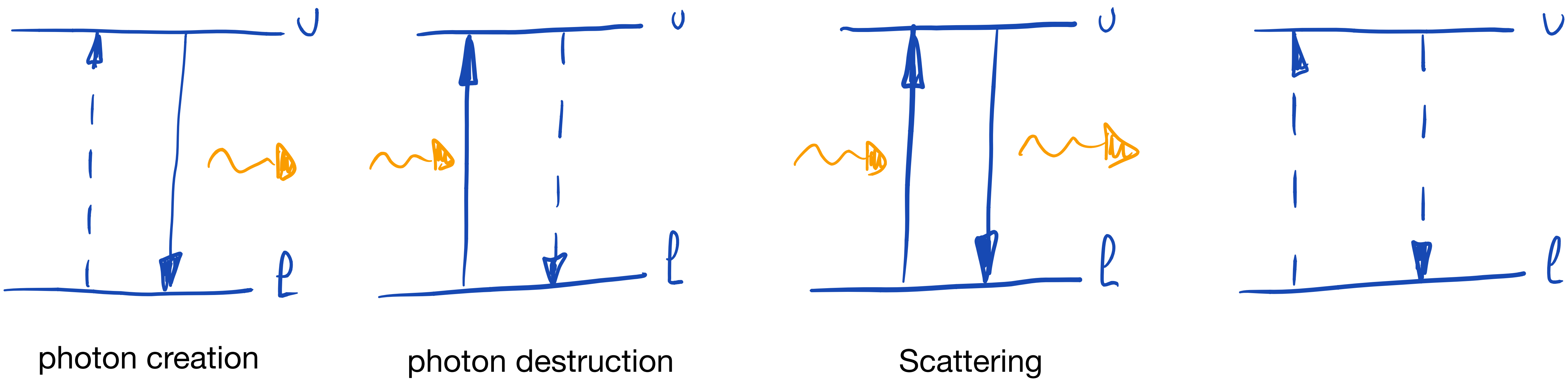
$$I_\nu \approx S_0 + S_1(1 - \eta_0 \psi(\nu - \nu_0))$$

The observed intensity profile is linearly dependent on ψ, and the width of the intensity profile is given by the Doppler width.

But in general, if the line is optically thick, the observed intensity profile is wider than ψ. **Opacity broadening**. So the line width cannot be related directly to temperature in optically thick case.

You can try to show it, by assuming a small value of η_0 and a large one. For example, $\eta_0 = 0.1$ and $\eta_0 = 15$ and assuming a Gaussian profile. Then measure the FWHM of I_ν!

Photon redistribution effects



Let's assume two situations for scattering atmospheres:

Coherent scattering: $R(\nu, \nu') = \delta(\nu' - \nu)$ photons are re-emitted with the same ν.

Complete redistribution: $R(\nu, \nu') = \phi(\nu)$, the frequency of the absorbed and emitted photons are completely uncorrelated.

Think how is the plasma in an atmosphere: **dynamic**
(thermal motions, macroscopic motions, collisions)

Which of the two regimes seems more realistic to you? Why?

Photon redistribution effects

Coherent scattering: $R(\nu, \nu') = \delta(\nu' - \nu)$ photons are re-emitted with the same ν.

The medium is optically thick, so photons can only **scatter many many times in order to escape**. They are re-emitted at the same frequency after each absorption but not necessarily in the same direction: “random walk”-like path.

The medium becomes less and less optically thick as we move in the vertical direction, so eventually photons can scape from the atmosphere.

Coherent scattering is a very inefficient way of escaping the atmosphere.
The thermalization depth occurs very high in the atmosphere: $\Lambda = 1/\sqrt{\epsilon}$

Photon redistribution effects

Complete redistribution: $R(\nu, \nu') = \phi(\nu)$, the frequency of the absorbed and emitted photons are completely uncorrelated.

Photons do not only scatter in direction, but **also in frequency**.

In most lines the wings become optically thin within a few Doppler widths.

If a photon is re-emitted in the optically thin wings, it can escape at once!

As you can imagine, collisions do help to uncorrelate the frequency of the absorbed and re-emitted photon.

If the profile is Lorentzian: $\Lambda = 1/\epsilon^2$

If the profile is Gaussian: $\Lambda = 1/\epsilon$

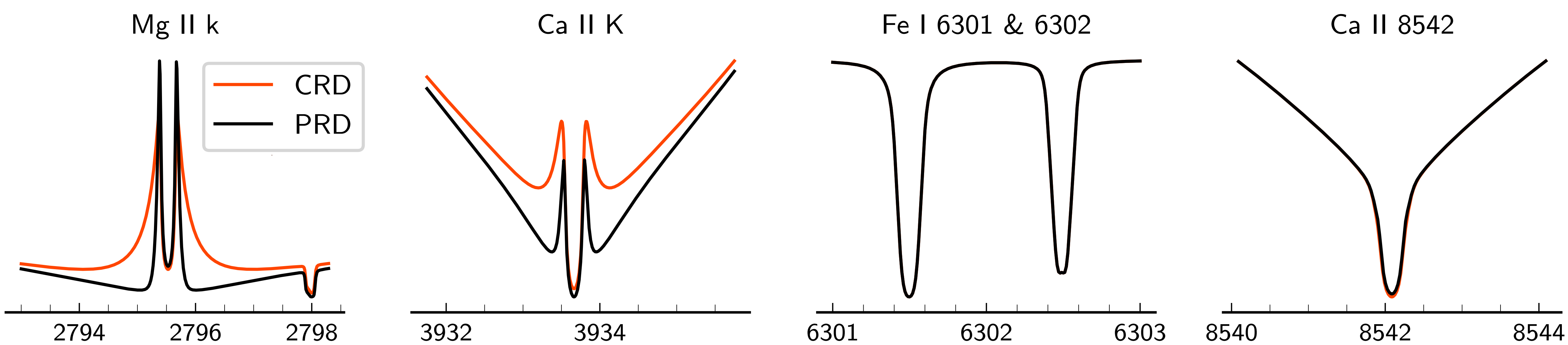
Thermalization occurs deeper in the atmosphere.

Complete redistribution is a much more efficient mechanism to escape via redistribution to the wings.

Partial redistribution effects

Now imagine an intermediate situation were re-emitted photons have some memory of the incoming frequency and direction.

This is the case of very strong lines were the damping wings are optically thick, so photons cannot escape as in complete redistribution and still have to scatter even at the wings.



In CRD: $\chi = \phi = \psi \longrightarrow S_\nu^{\rm line} = \dfrac{n_u A_{ul}}{n_l B_{lu} - n_u B_{ul}}$

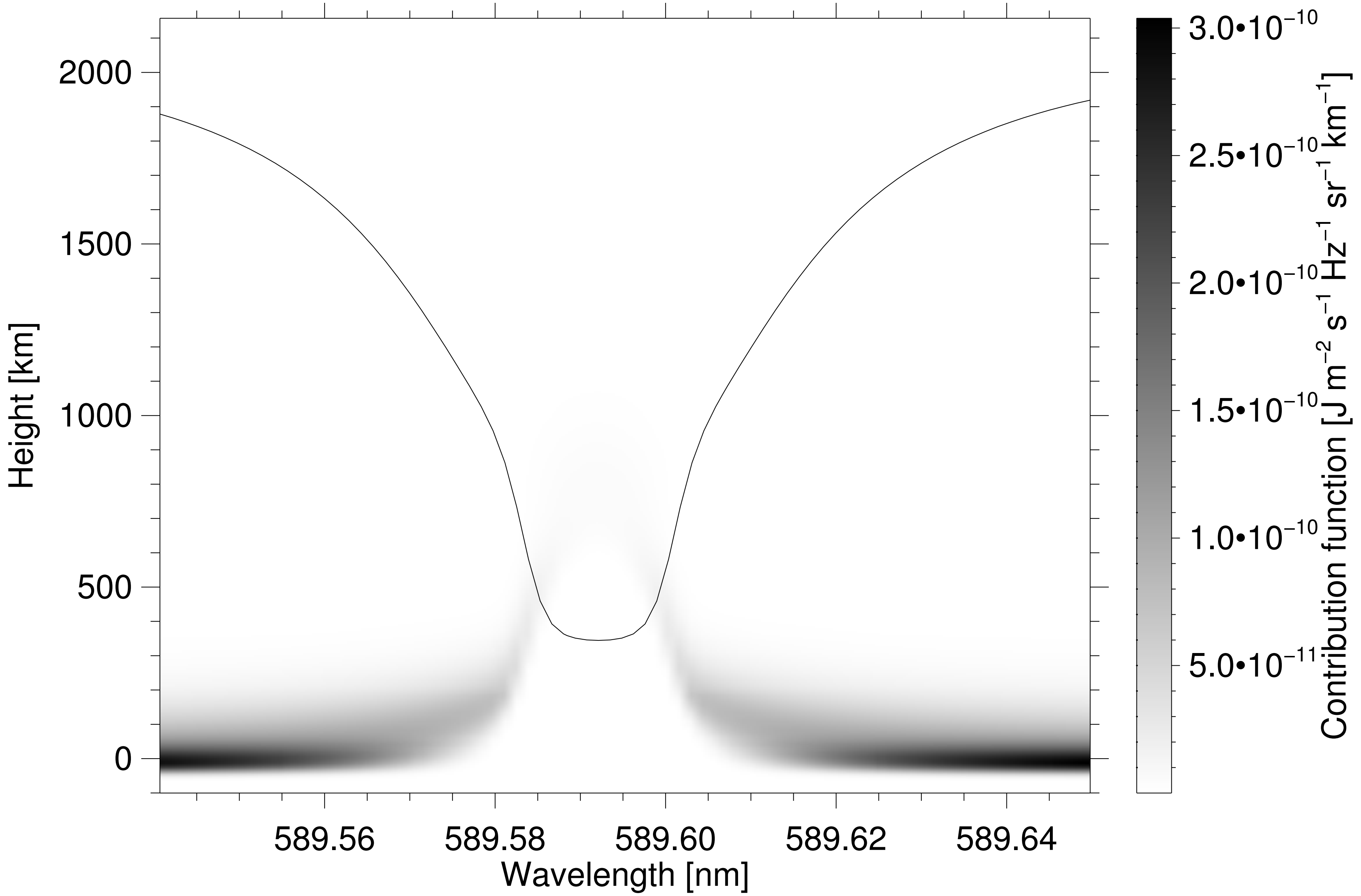
In PRD: $\chi \neq \phi \neq \psi \longrightarrow S_\nu^{\rm line} = \dfrac{n_u A_{ul} \psi}{n_l B_{lu} \phi - n_u B_{ul} \chi}$

Sensitivity of the line: where is it formed?

Perhaps the right question is: where is it formed **at which wavelength?**
Contribution functions **do not contain information of the non-local part.**

$$I_\nu = \int_{-\infty}^{\tau_\nu^0} S_\nu(z) e^{-\tau_\nu(z)} \frac{d\tau_\nu(z)}{dz} dz$$

C_ν = **contribution function**



Reproduced from Uitenbroek (2006)

Sensitivity of the line: where is it formed?

Response functions are more useful, because they relate to real physical parameters:

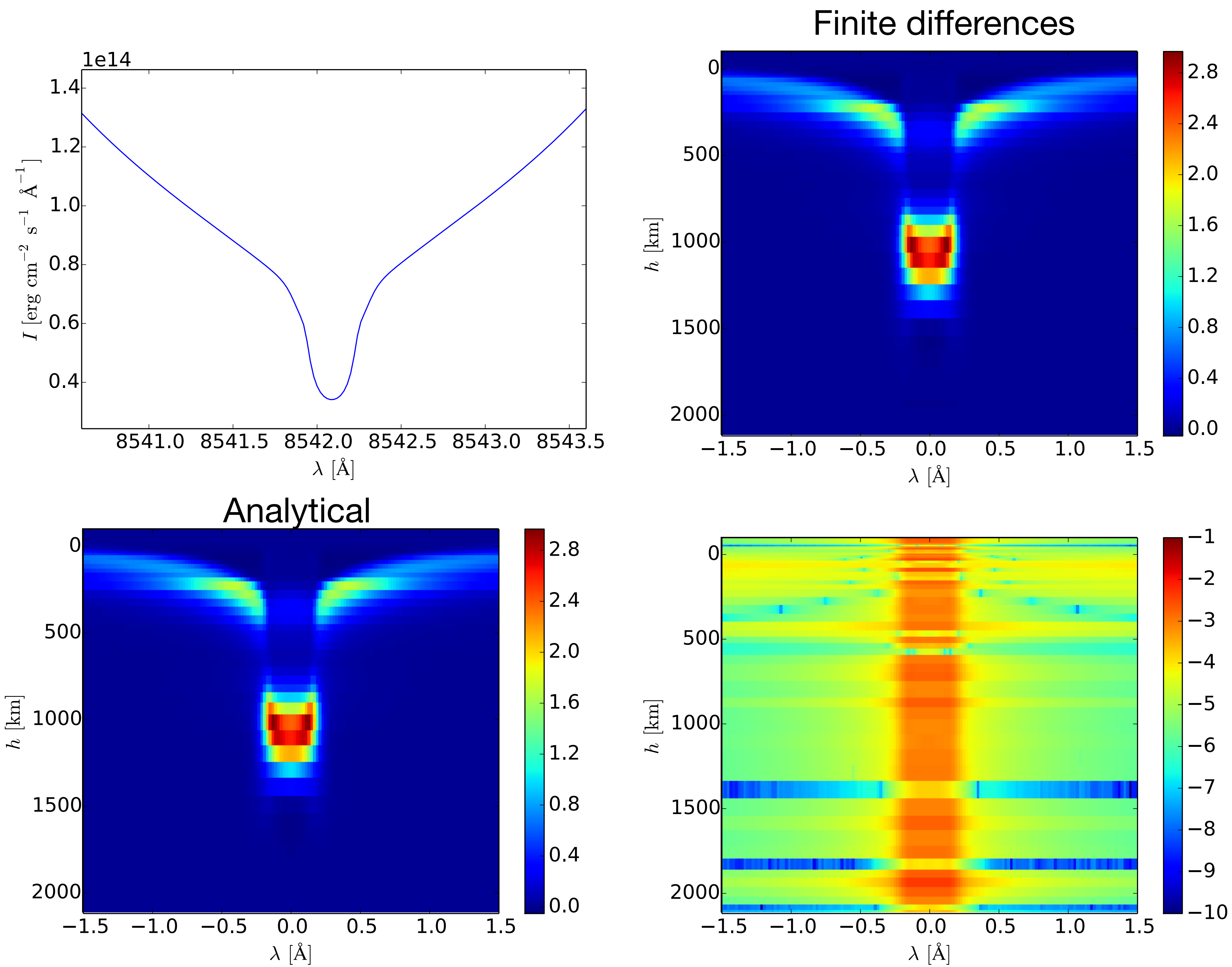
$$R_\nu^X(z) = \frac{\delta \boldsymbol{I}_\nu(z, X)}{\delta X \delta z}$$

The allow to compute the sensitivity with respect to each physical parameter:

$$T, v_{l.o.s}, v_{turb}, \boldsymbol{B}$$

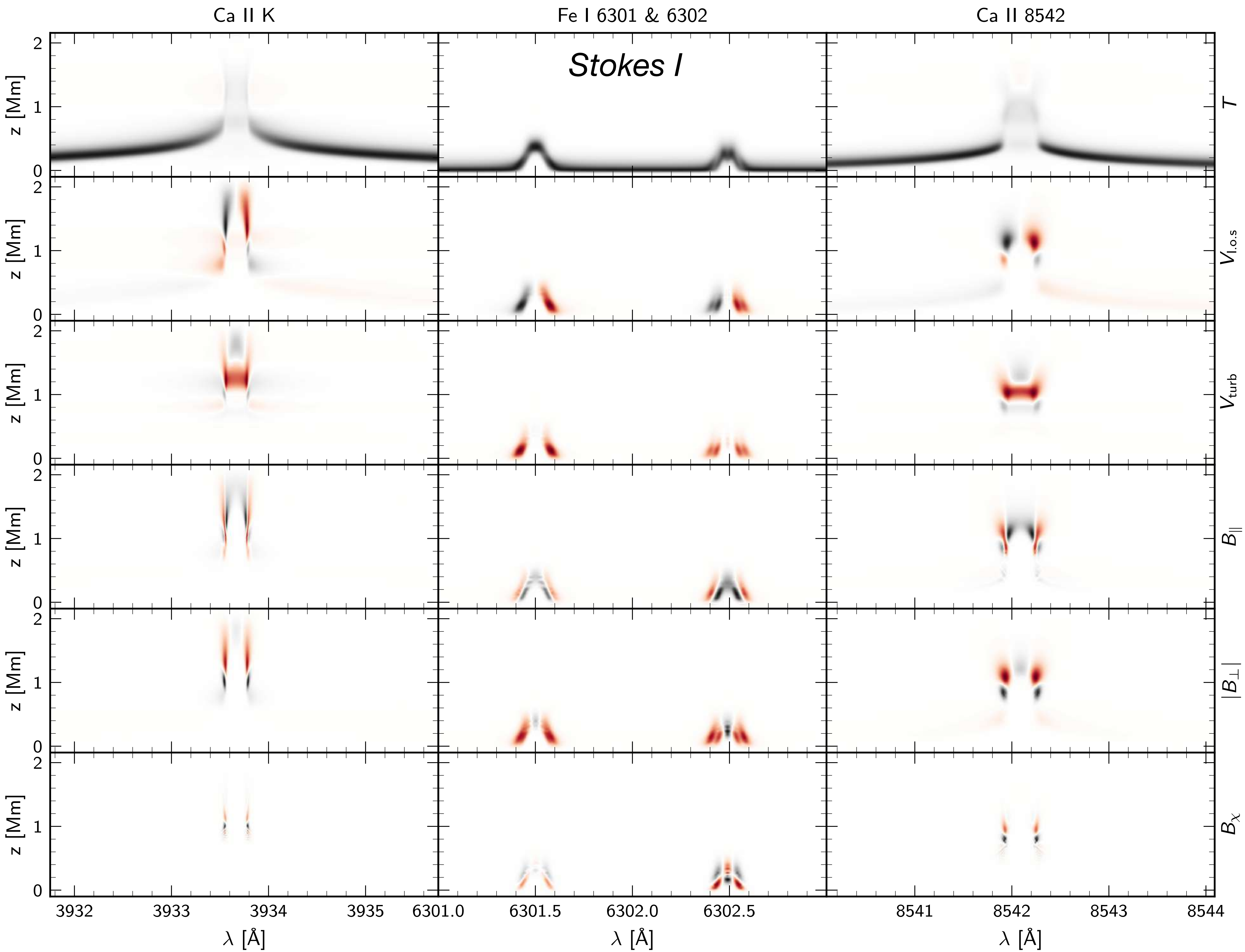
Traditionally we could only compute response functions analytically in LTE (e.g., the SIR code). But recently Milic & van Noort (2017) showed that it is possible to compute NLTE CRD response functions

Sensitivity of the line: where is it formed?

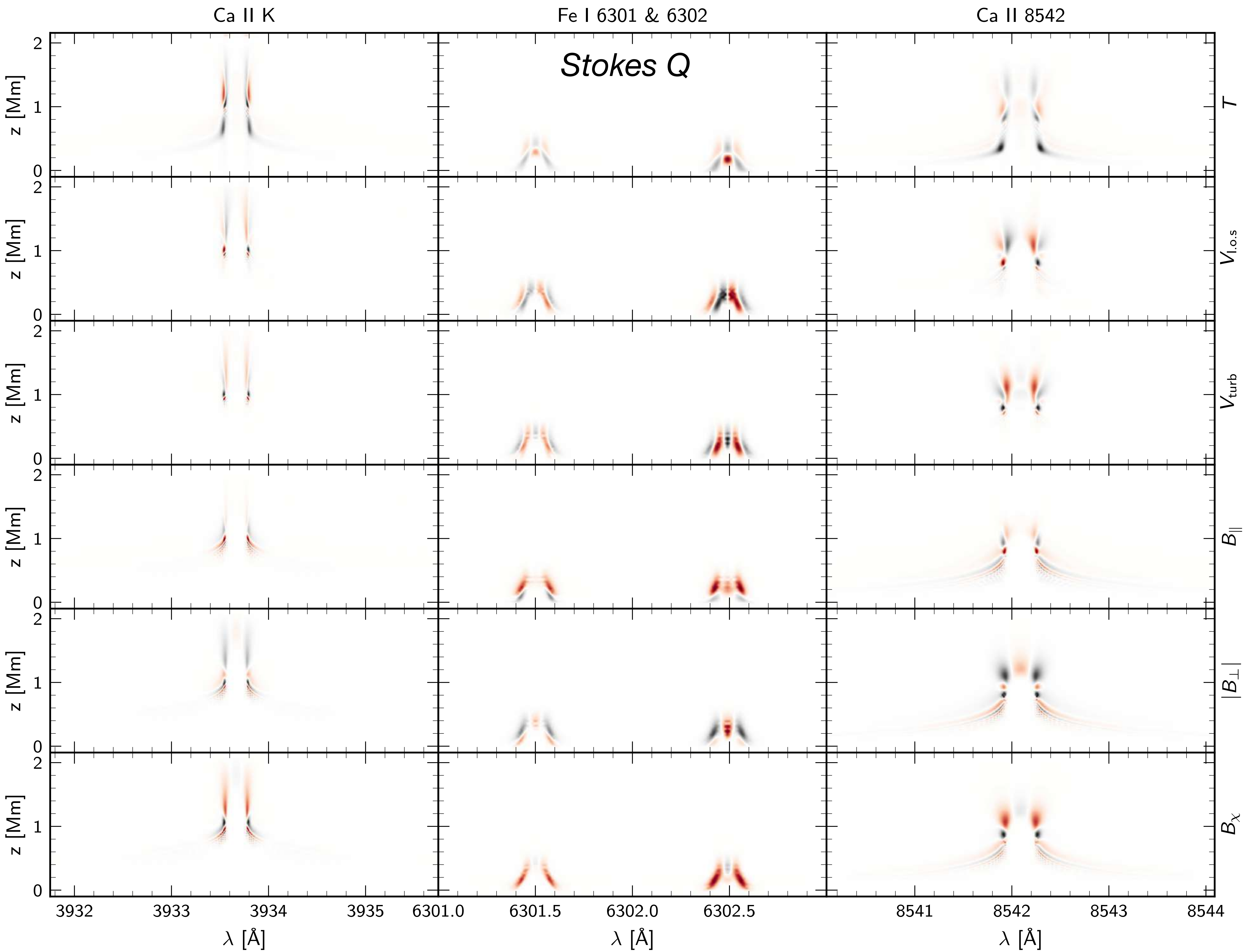


Milic & van Noort (2017)

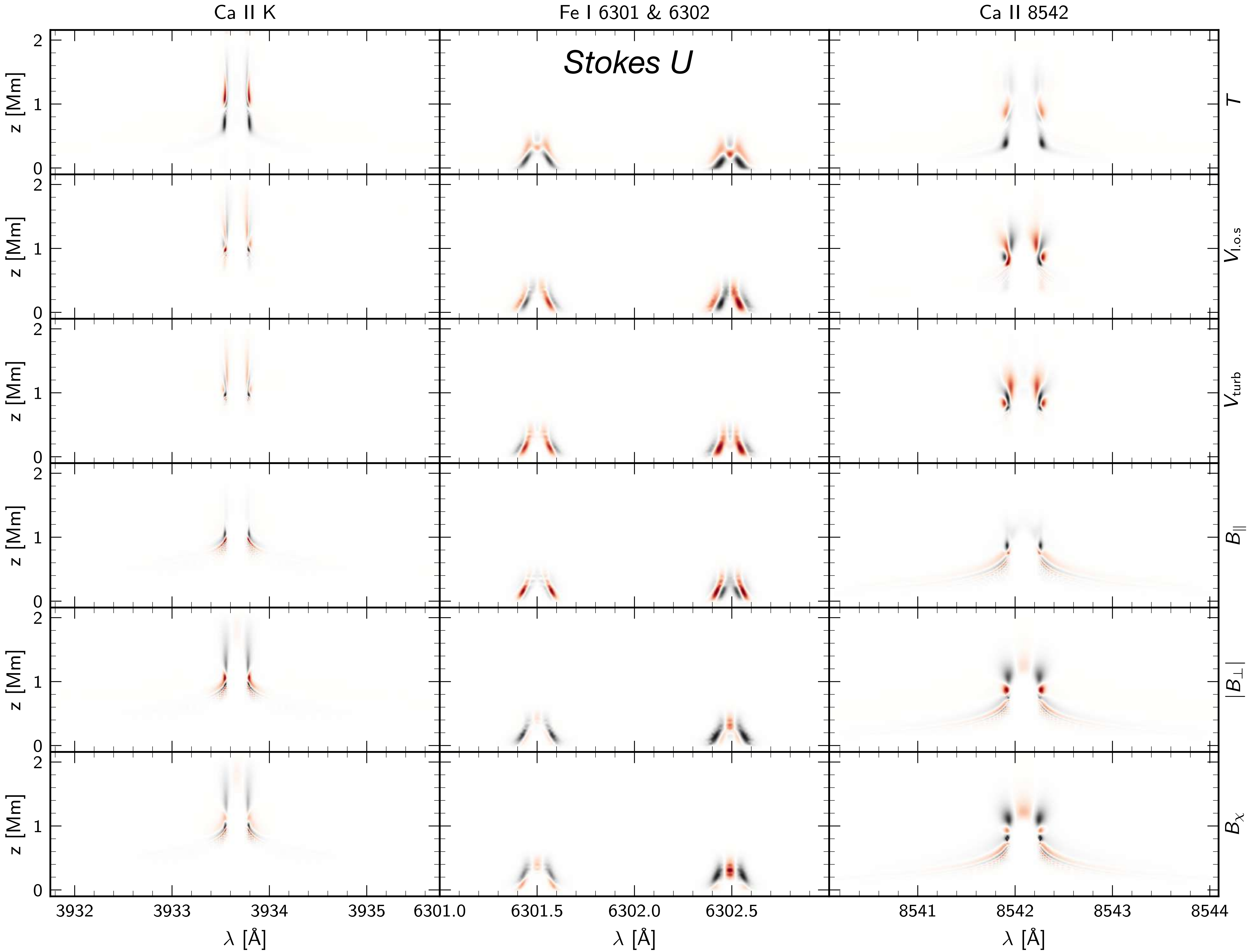
Sensitivity of the line: where is it formed?



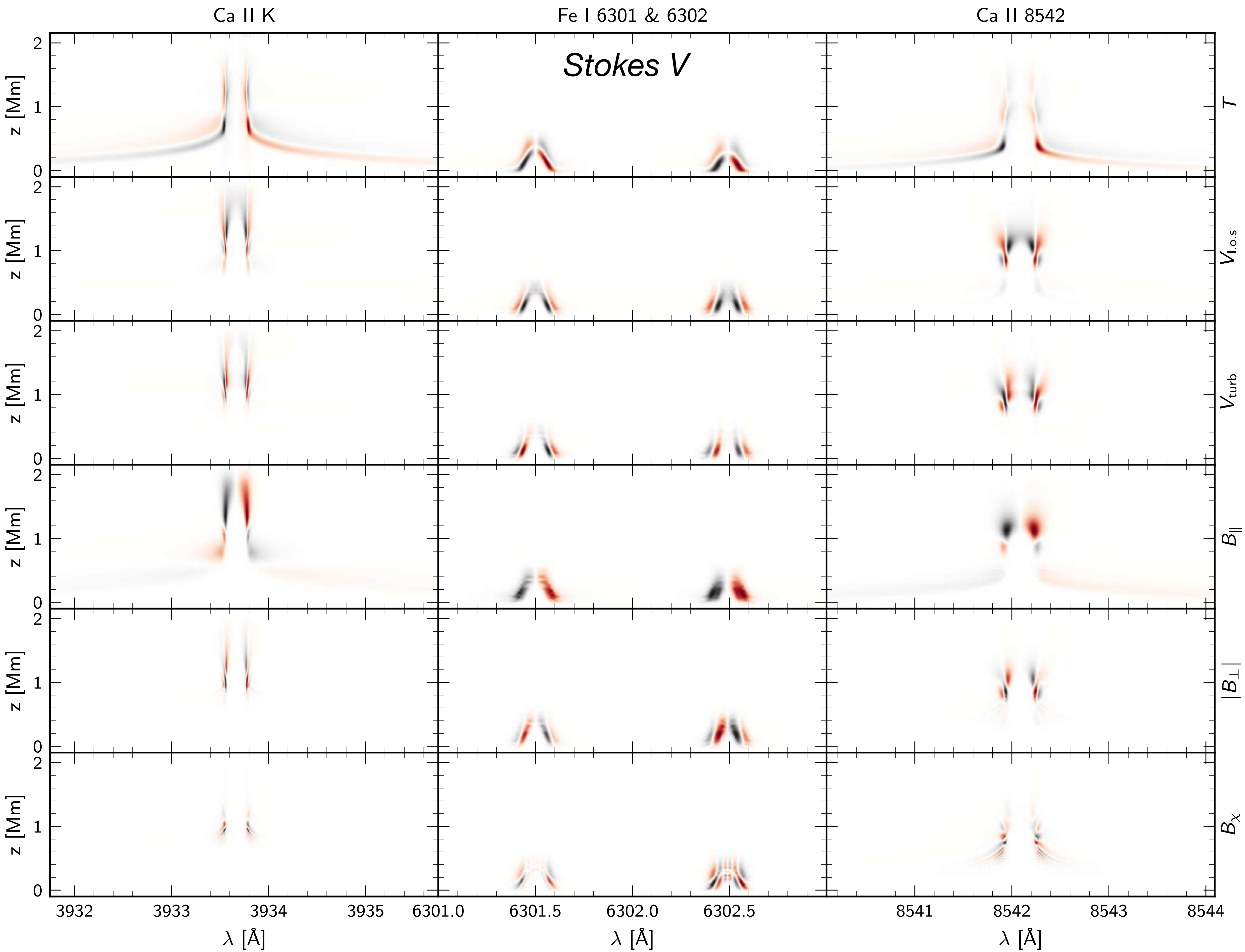
Sensitivity of the line: where is it formed?



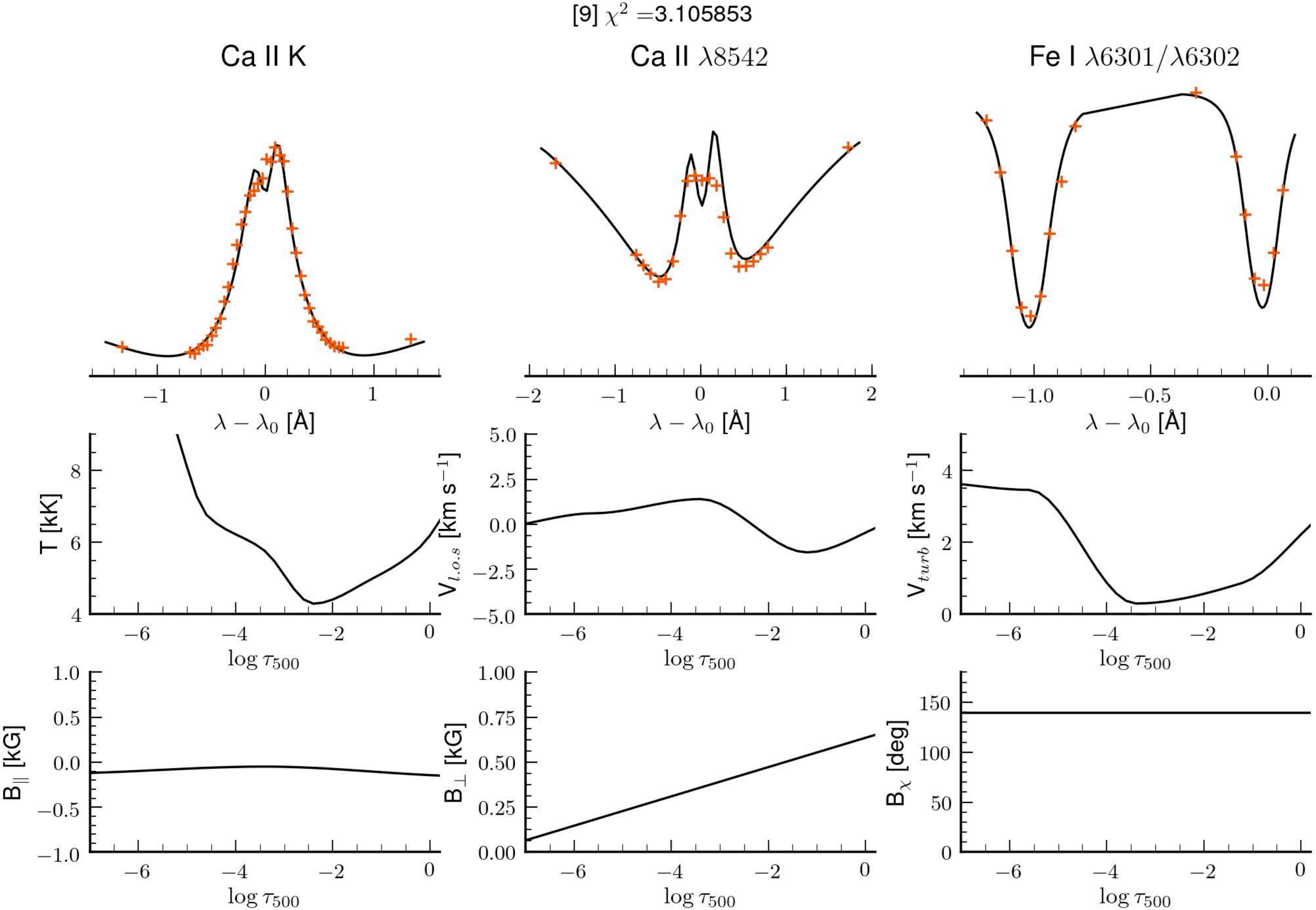
Sensitivity of the line: where is it formed?



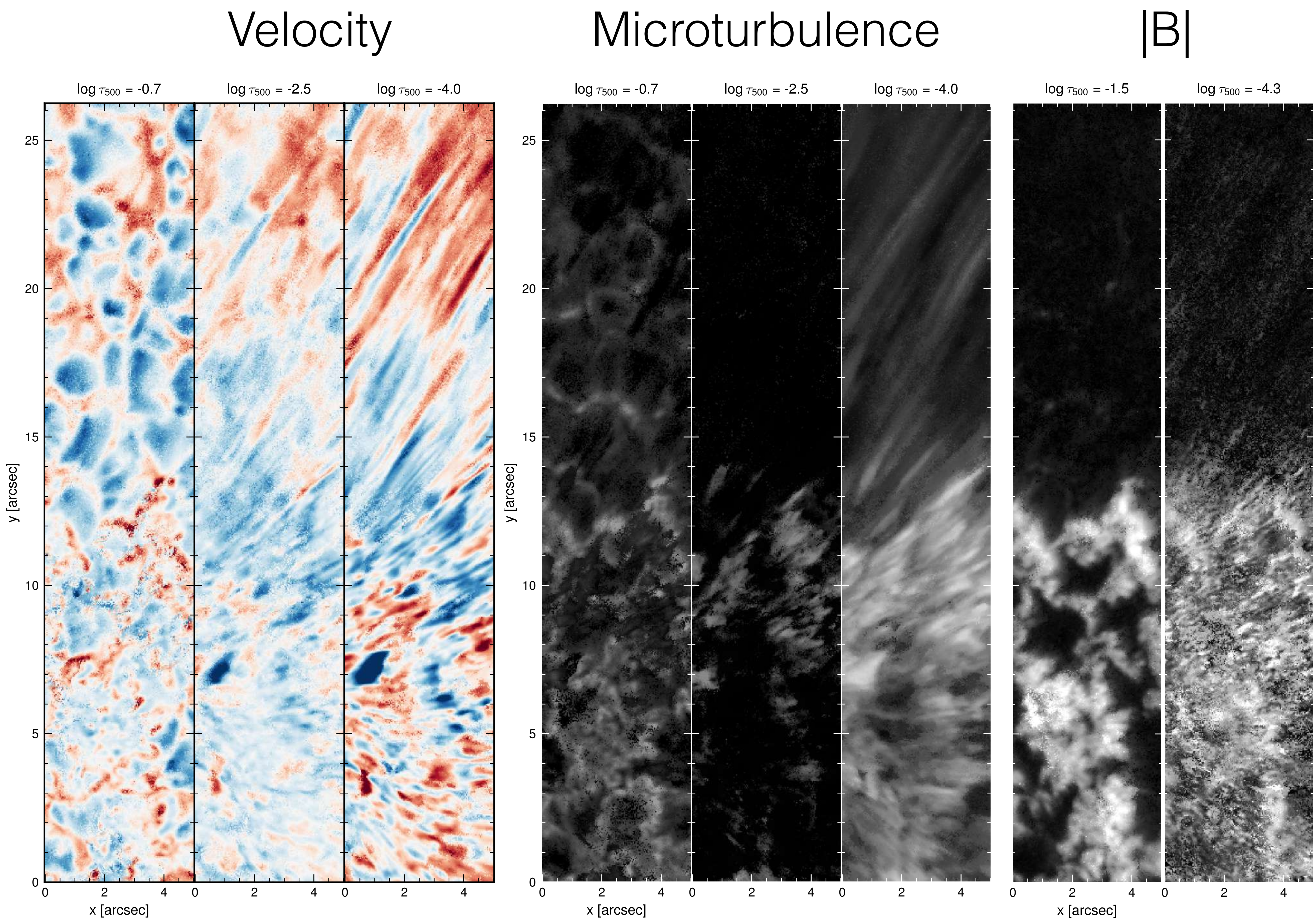
Sensitivity of the line: where is it formed?



Response functions in data modelling: inversions

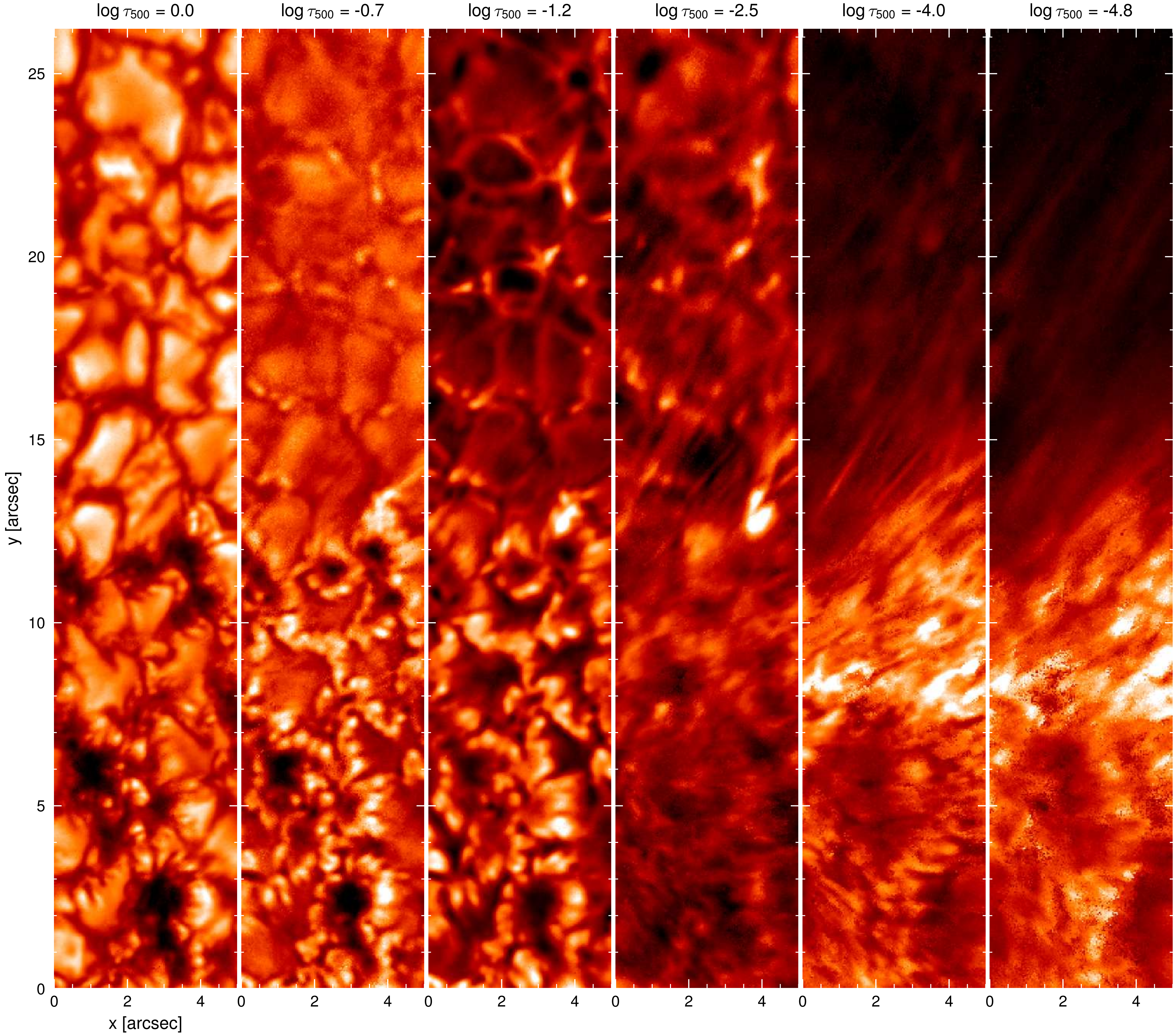


NLTE inversions



NLTE inversions

Temperature



To take home

- Partial redistribution effects are important in very strong lines where the damping wings are optically thick.
- Partial redistribution is an intermediate case between coherent scattering and complete redistribution of scattered photons.
- Contribution functions provide information of the formation of the line but they only depend on the opacity and the source function.
- Response functions allow estimating the sensitivity of spectral lines at each wavelength to changes in the physical parameters of the model atmosphere.
- Response functions allow performing NLTE inversions based on gradient descent methods (like the Levenberg-Marquardt algorithm).